Specific Impulse of a Liquid-Core Nuclear Rocket

W. Louis Barrett Jr.*
Boeing Company, Seattle, Wash.

Nomenclature

A = liquid fission fuel surface area, ft^2
F = rocket thrust, lbf
g = gravitational conversion constant = 32.2 (lb/lbf) (ft/sec^2)
I = specific impulse, lbf-sec/lbm
M_f = fission fuel molecular weight
M_H = propellant molecular weight
$\langle M \rangle$ = average molecular weight in core
P_a = actual fission fuel partial pressure in core, lbf/ft^2
P_c = total gas pressure in core, lbf/ft^2
P_f = fission fuel vapor pressure, lbf/ft^2
P_H = propellant partial pressure in core, lbf/ft^2
R = gas constant, 1.5×10^3, ft-lb/lbm-°R
T = gas temperature in core, °R
γ = specific heat ratio

A NUCLEAR rocket engine that operates with its fission fuel in the molten state (Fig. 1) has often been mentioned as a method of obtaining specific impulses higher than those available to the solid-core nuclear rocket.[1] This note discusses the specific impulse that might be expected from such an engine. Another limitation, heat transfer, is presently under investigation but will not be discussed here.

The ideal specific impulse of a propellant exhausting through an adiabatic nozzle is

$$I = \left[\frac{2\gamma}{\gamma - 1}\frac{R\,T}{g\,\langle M \rangle}\right]^{1/2} \quad (1)$$

The average molecular weight of a two-component gas mixture is

$$\langle M \rangle = (P_f/P_c)M_f + (P_H/P_c)M_H \quad (2)$$

or, since

$$P_c = P_f + P_H$$
$$\langle M \rangle = (1/P_c)[P_cM_H + P_f(M_f - M_H)] \quad (3)$$

The temperature dependence of the fuel vapor pressure can be obtained from the Clausius-Clapeyron equation. The result is

$$P_f = \exp[-(B/T) + C] \quad (4)$$

where B and C are constants for a given substance.

The vapor pressure increases by a factor of 1000 to 10^6 for the various materials shown in Fig. 2 as the core temperature is increased from 5000° to 10,000°R. The vapor pressure increases so rapidly that below a certain value of core temperature the vapor pressure has almost no effect on the specific impulse, and above this temperature the vapor pressure is the most important term, actually reducing the specific impulse as core temperature is increased.[4–7]

Figure 3 is a plot of specific impulse vs core temperature for several materials. These assume that 1) chamber pressure is 1000 psi (specific impulse increases slightly with chamber pressure); 2) constant specific heat ratio; 3) liquid fuel temperature equals core gas temperature; and 4) molecular weight of the fuel vapor is the same as that of the liquid fuel.

The last assumption may not be true if the fuel is a compound. A rapid drop in specific impulse occurs when the fuel partial pressure becomes a small fraction of the core pressure. If the temperature is increased beyond the boiling temperature (vapor pressure equals 1000 psi), the specific impulse renews its characteristic increase as temperature is increased. The thorium compounds in Fig. 3 probably cannot be used by themselves, since it appears that thorium will not sustain a chain reaction.[2]

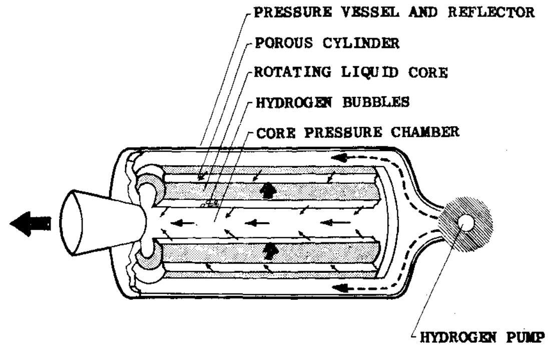

Fig. 1 Liquid core nuclear rocket: bubbling concept.

There are a few possible methods of decreasing the fuel vapor pressure. One of these is to alloy the fissionable material with another substance. This sometimes produces startling changes in the vapor pressure with only small amounts of the alien metal. Another possibility is to float a less dense liquid with low vapor pressure on top of the fission fuel. This technique will work only if the buildup of vapor in the bubbles, while traversing the fission fuel, is much smaller than that vapor produced at the liquid surface in the reactor pressure chamber.

Another scheme (Fig. 4) being considered would use fission fuel (either gas or liquid core reactor) to melt a material such as tungsten or hafnium carbide. Propellant would be heated by spraying the molten material into the propellant. A molten tungsten system would have a specific impulse similar to ThO_2.

The maximum temperature of the liquid core is absolutely restricted to a value below the critical temperature of the fuel material, since above the critical temperature substance cannot exist in the liquid state. The critical temperature for uranium is about 22,000°R, and other fission materials

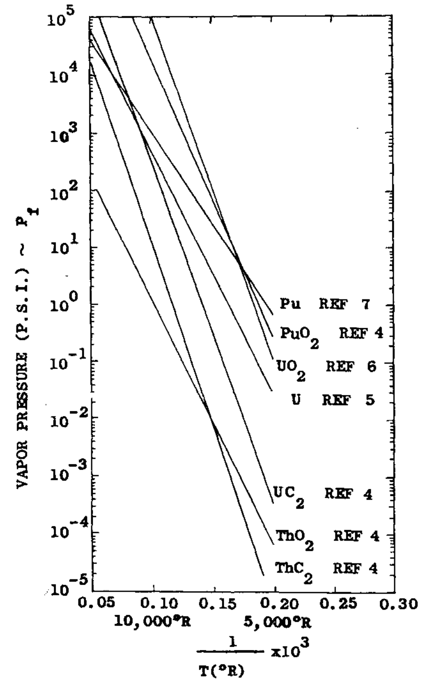

Fig. 2 Vapor pressures of several fission fuels.

Received July 25, 1963.
* Associate Research Engineer, Flight Technology, Aero-Space Division.

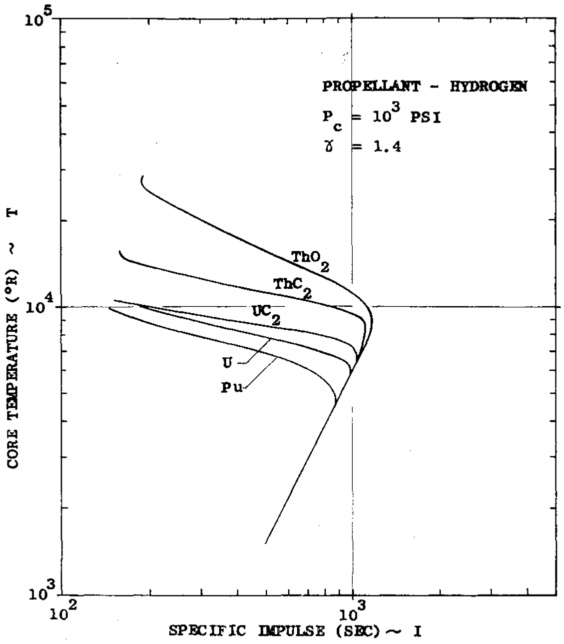

Fig. 3 Specific impulse vs core temperature for several materials.

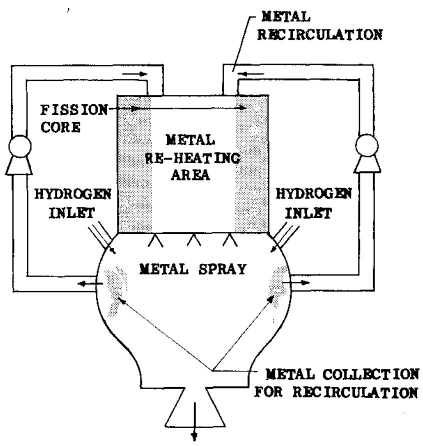

Fig. 4 Liquid-core nuclear rocket: spray concept.

are of this order; therefore, this restriction will probably be only of academic interest.

It has been assumed in these calculations that the partial pressure of fission fuel in the rocket pressure chamber is equal to the liquid's vapor pressure. This may not be the case if the liquid surface is incapable of producing vapor at the rate fission fuel is exhausted through the nozzle.

Kinetic theory yields an expression for the rate matter leaves the liquid surface[3]:

$$\frac{dW_a}{dt} = A(P_f - P_a)\left[\frac{M_f g}{2\pi RT}\right]^{1/2}$$

where A is the fission fuel surface area plus an area term due to the bubble surfaces, and P_a is the actual fuel partial pressure.

The rate mass is exhausted from the nozzle:

$$\frac{dW_f}{dt} = \frac{\text{total fission fuel exhausted through nozzle}}{\text{rocket operating time}}$$

This may be expressed as

$$\frac{F}{I}\,\frac{P_a M_f}{P_c M_H + P_a(M_f - M_H)}$$

The ratio of dW_a/dt to dW_f/dt is

$$\frac{AP_c}{F}\left[\frac{\gamma\langle M\rangle}{(\gamma - 1)\pi M_f}\right]^{1/2}\left[\frac{P_f}{P_a} - 1\right]$$

At equilibrium vapor pressure, this ratio equals one. An estimate of P_f/P_a may be made by assuming that, if $M_f \sim 10^2$, $\langle M\rangle \sim 2$, $A \sim 10$ ft^2, $P_c \sim 10^3$ psi, $F \sim 10^5$ lbf, then $P_a = \frac{1}{2}P_f$. Since the maximum specific impulse occurs when P_f/P_c is about 10^{-3}, the use of P_f vice P_a will make a negligible difference in the specific impulse maximum.

Several materials and engine designs have been considered, and it appears that the maximum specific impulse of the liquid-core nuclear rocket will be in the range of 1200 to 1400 sec.

References

[1] Rom, F. E., "Advanced reactor concepts for nuclear propulsion," Astronautics 4, 20–22 (October 1959).

[2] Glasstone, S., *Nuclear Reactor Engineering* (D. Van Nostrand Co. Inc., Princeton, N. J., 1955), 1st ed., p. 39.

[3] Kennard, E. H., *Kinetic Theory of Gases* (McGraw-Hill Book Co. Inc., New York, 1958), p. 63.

[4] Vaughan, D. A., "Reactor Core Materials," Battelle Memorial Inst., 4, no. 4, 13 (1961).

[5] Rauh, E. G. and Thorn, R. J., "Vapor pressure of uranium," J. Chem. Phys. 22, 1414–1420 (1954).

[6] Katz, J. J. and Rabinowitch, E., *Chemistry of Uranium* (Dover Publications Inc., New York, 1951), 1st ed., p. 262.

[7] Finnston, H. M. and Howe, J. P., "Metallurgy and fuels," *Nuclear Energy Series* (Pergamon Press, New York, 1956), Vol. 1, no. 5, p. 384.

A Uniqueness Theorem for the Nonlinear Axisymmetric Bending of Circular Plates

Edward L. Reiss*
Courant Institute of Mathematical Sciences, New York University, New York, N. Y.

I. Introduction

In this note, the nonlinear axisymmetric bending of circular plates is considered within the scope of the von Kármán theory.[1] It is assumed that the plate is deformed by a symmetrically distributed pressure applied normal to one face. For a variety of boundary conditions along the edge of the plate, the von Kármán equations reduce to a coupled pair of second-order nonlinear ordinary differential equations. The uniqueness theorem given in Ref. 2 for the solution of one of these boundary value problems is valid only for a limited range of parameters. Morosov's[3] uniqueness theorem is established with the aid of the Hildebrand-Graves theorem.

In this paper, an "elementary" proof is given of the uniqueness of the symmetric solutions of von Kármán's equations. For simplicity, only one set of boundary conditions is considered. However, with suitable modifications, uniqueness for other boundary conditions can also be proved. First, the boundary value problem is cast in a form similar to that used by Friedrichs and Stoker[4] for the buckling of circular plates. The uniqueness proof then follows directly from the form of the potential energy functional.

Received July 30, 1963. This paper represents results obtained at the Courant Institute of Mathematical Sciences, New York University, under the sponsorship of the U. S. Army Research Office (Durham) Grant No. DA-ARO-(D)-31-124-G344. Reproduction in whole or in part is permitted for purposes of the United States Government.

* Associate Professor of Mathematics.